Member's Default Utility Function for Default Fund Design Version 1 ("MDUF v1")

Technical Paper No.2: Static Models

David Bell[*], Estelle Liu[†], Dr. Adam Shao[‡]

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1 Introduction

This paper provides technical detail into the application of MDUF v1 to static retirement solutions. It should be read together with the Excel file titled "Static Models - MDUF v1.xlsb" ("Excel Model"), following a review of the "Introduction Paper" and "Technical Paper No.1: MDUF v1 Design". Note that the numerical results provided in this paper can be different between those in the Excel Model, as the simulations used in the Excel Model are updated each time calculations are rerun.

As discussed in the "Introduction Paper", providing retirement outcome solutions is a hugely challenging and complex area. Technically the retirement outcomes problem can be defined as:

"A dynamic, integrated consumption and investment decision problem."

A dynamic solution accounts for both current market conditions and personal situation and is evaluated on a regular basis, e.g. every year. A static solution is a set of predetermined rules applied throughout retirement. Solution techniques required in a dynamic solution are more complex than those in a static solution.

In this paper we provide greater detail, in a more complex setting compared to the "Introduction Paper", around the application of MDUF v1 to comparing static retirement solutions. For dynamic solutions, please refer to "Technical Paper No.3: Optimal Dynamic Strategies".

[*]David Bell is a lead author of MDUF v1, the Chief Investment Officer at Mine Wealth and Wellbeing and a PhD candidate at UNSW.

[†]Estelle Liu is a lead author of MDUF v1, and a Quantitative Analyst (Retirement Outcomes) at Mine Wealth and Wellbeing.

[‡]Dr. Adam Shao is a lead author of MDUF v1, a Senior Quantitative Analyst (Retirement Outcomes) at Mine Wealth and Wellbeing and an Associate Investigator at CEPAR (ARC Centre of Excellence in Population Ageing Research).

2 Building Up the Framework in Detail

2.1 Key Variables

In comparing static retirement solutions, the utility function is designed in a way to capture the joint effect of drawdown rules (which determine consumption levels) and asset allocations. This can be represented by O_t which denotes the vector that consists of choice variables at time t including drawdown and asset allocation, i.e.

$$O_t = (c_t,\ \omega_t) \quad 0 < t \leq T, \tag{2.1}$$

where c_t is the annual consumption level at time t according to a predetermined strategy (including a drawdown rule, asset annuitisation, and the impact of the Age Pension), and ω_t is the percentage weight of wealth invested in risky asset at time t.

Let a denote the annuitisation ratio, which is a decision made at retirement. Let b_t denote the wealth dynamics. The annuitisation ratio multiplied by the initial wealth is equal to the amount of wealth invested into life annuities. Let L denote the price of a unit life annuity that pays \$1 p.a. throughout retirement. Let l_t denote the annual amount of benefit from life annuities, we have

$$l_t = \frac{b_0 a}{L}. \tag{2.2}$$

The dynamics of wealth is

$$\begin{aligned} b_1 &= (b_0 - b_0 a + P_0 + l_0 - c_0)\left(1 + \tilde{R}_1\right), \\ b_{t+1} &= (b_t + P_t + l_t - c_t)\left(1 + \tilde{R}_{t+1}\right), \quad \text{for } t > 0 \end{aligned} \tag{2.3}$$

where P_t is the amount of Age Pension entitlement received at time t if considered, and $\tilde{R}_t$ is the stochastic return p.a. of the investment portfolio from time $t-1$ to t, which is assumed to follow a Normal distribution.

2.2 Utility Function

$u(c_t)$ is the utility function defined over consumption at time t:

$$u(c_t) = \frac{c_t^{1-\rho}}{1-\rho}. \tag{2.4}$$

$v(b_t)$ is the utility function defined over end-of-life residual benefit:

$$v(b_t) = \frac{b_t^{1-\rho}}{1-\rho}\left(\frac{\phi}{1-\phi}\right)^{\rho}, \tag{2.5}$$

where b_t is the level of wealth at time t which equals the amount of residual benefit if the person dies between $t-1$ and t. $\rho > 0$ is the level of risk aversion; higher ρ means greater risk aversion of the investors. ϕ is the strength of residual benefit motive; higher ϕ means stronger residual benefit motive.

2.3 Expected Lifetime Utility

The expected lifetime utility function (i.e. our MDUF v1), taking into account mortality, is therefore:

$$U_0 = \mathbb{E}_0 \left[\sum_{t=0}^{T} \left\{ {}_t p_x \ u(c_t) + {}_{t-1|}q_x \ v(b_t) \right\} \right], \tag{2.6}$$

where x is the inception age of a particular cohort, T is the retirement planning horizon ($x + T$ is the maximum age), $\mathbb{E}_0$ is the expectation operator with respect to time 0 (equivalently, age x), ${}_tp_x$ is the probability of being alive at age $x + t$ conditional on being alive at age x, and ${}_{t-1|}q_x$ is the probability of dying between age $x + t - 1$ and $x + t$ conditional on being alive at age x.

The expected lifetime utility calculated based on different retirement strategies (e.g. different drawdown rules, different investment portfolios) and/or having various retirement products (e.g. life annuities) can then be compared.

2.4 The Age Pension

2.4.1 Rules for account-based pension

The amount of Age Pension entitlement is means-tested, i.e. depending on the current asset value and future incomes[1]. We formulate the following tests to determine the Age Pension entitlement:

- Asset test

  $$P_t^A = \max\left(\bar{P} - \tau^A \max\left(b_t - \underline{b},\ 0\right),\ 0\right), \tag{2.7}$$

  where P_t^A is the Age Pension entitlement under the asset test, $\underline{b}$ is the asset test threshold for full pension, $\bar{P}$ is the full Age Pension payment rate, and τ^A is the taper rate for asset test.

- Income test under deeming rule

  $$P_t^I = \max\left(\bar{P} - \tau^I \max\left(r_1 \min\left(b_t,\ \underline{b}_1\right) + r_2 \max\left(b_t - \underline{b}_1,\ 0\right) - \underline{I},\ 0\right),\ 0\right), \tag{2.8}$$

  where P_t^I is the Age Pension entitlement under the income test, $\underline{b}_1$ is the deeming threshold, below which a lower deeming rate r_1 is applied and above which a higher deeming rate r_2 is applied. $\underline{I}$ is the income test cut off point. τ^I is the taper rate for income test.

- Combining the two tests

  $$P_t = \min\left(P_t^A,\ P_t^I\right). \tag{2.9}$$

2.4.2 Rules for life annuities

Based on the Age Pension eligibility rules for lifetime income streams, we formulate the following tests to determine the Age Pension entitlement:

[1] See the Australia Government's Age Pension website at `https://www.humanservices.gov.au/customer/services/centrelink/age-pension`

- Asset test

$$P_t^A = \max\left(\bar{P} - \tau^A \max\left(b_t^p + b_t^a - \underline{b},\ 0\right),\ 0\right), \tag{2.10}$$

  where

$$b_t^a = \max\left(b_0^a - \frac{b_0^a}{\mathring{e}_x} \times t,\ 0\right), \tag{2.11}$$

  b_t^p and b_t^a are the time t value of assets in account-based pension and annuity respectively. b_0^a is the purchase price of annuity at time 0. $\mathring{e}_x$ is the life expectancy of the member age x when the annuity is purchased at time 0.

- Income test

$$P_t^I = \max\left(\bar{P} - \tau^I \max\left(r_1 \min\left(b_t^p,\ \underline{b}_1\right) + r_2 \max\left(b_t^p - \underline{b}_1,\ 0\right) + \max(I_t^a - \frac{b_0^a}{\mathring{e}_x},\ 0) - \underline{I},\ 0\right),\ 0\right), \tag{2.12}$$

  where I_t^a is the annuity income at time t and $\frac{b_0^a}{\mathring{e}_x}$ is the deduction amount under the Age Pension rule for annuity.

- Combining the two tests

$$P_t = \min\left(P_t^A,\ P_t^I\right). \tag{2.13}$$

2.5 Expected Lifetime Utility Calculation

In the initial period, the wealth is divided into annuitised and non-annuitised parts. Annuitised wealth will be converted into income streams for the remaining lifetime based on the life annuity prices.

For each of the following periods, the level of consumption is determined based on predetermined draw-down rules and eligible Age Pension entitlements. For example, based on the minimum draw-down rule, the draw-down amount is calculated as a predetermined proportion of wealth. After subtracting the consumption and adding annuitised income (if any), the remaining non-annuitised wealth will grow at a stochastic rate that depends on the equity market as well as the investment strategies. Equity returns are modelled using Monte-Carlo simulations.

For each period and each path of simulated equity returns, we calculate the utility of consumption if the individual survives and the utility of residual account if the individual dies. The lifetime utility for each simulated path is calculated as survival probability weighted lifetime utility, enabling us to incorporate the distribution of mortality outcomes. The expected lifetime utility is the average lifetime utility across these simulated paths.

An alternative approach is to simulate both investment returns and mortality outcomes, effectively simulating individual retirement outcomes for each path of investment performance and mortality. The expected lifetime utility is the average lifetime utility across these two-dimensional simulation paths. This approach is intuitive but less efficient. It requires a large number of simulations to achieve the same level of accuracy as the survival probability weighted approach. We use the survival probability weighted approach in the Excel Model.

Another useful measure related to lifetime utility is Certainty Equivalent Consumption (CEC), which is calculated as the consumption level in the one-period utility function, i.e. Equation (2.4), that equates the utility level to the expected lifetime utility. CEC, in essence, is a monotonic transformation of the expected lifetime utility. A higher level of expected lifetime utility also corresponds to a higher CEC level. Note that CEC does not necessarily convey information of the actual level of consumption.

3 A Worked Example in Detail

We work through a comparative study of expected lifetime utility for different static retirement strategies. Mortality rates are sourced from the Australian Life Tables 2010-12 by Australian Government Actuary[2].

3.1 Four Cases

We compare four cases where the draw-down rules and annuitisation strategy are different. In Case 1, all assets are converted into life annuity; In Case 2, the minimum drawdown rule is applied on account-based pension (ABP); Case 3 has a constant target income of $ 43,372 p.a. with only ABP; Case 4 has a constant target income of $ 43,372 p.a. with 50% ABP and 50% life annuity. Table 3.1 summarises the four cases in our study.

Table 3.1: Description of different cases.

Case Number	**Description**
Case 1	100% Life Annuity
Case 2	Minimum drawdown rule on ABP
Case 3	Target constant income stream on ABP
Case 4	Target constant income stream (Life Annuity + ABP)

Note that the same investment strategy is used in these four cases, i.e. 50% in risk-free asset and 50% in equity.

3.2 Parameter Values and Input

Parameter values for preference and financial assets used in this paper are shown in Table 3.2. Note that in the Excel model there is more flexibility around changing these parameter values and inputs, following the instructions below:

- The first four parameters in Table 3.2 should be determined based on the market conditions and the investment characteristics of the fund portfolio. "Excel Model" users can change these values based on their views on the financial market and fund portfolios.
- The values for the fifth and sixth parameters in Table 3.2 are selected based on academic literature and empirical calibrations on members' risk preference. These are the recommended parameter values as part of MDUF v1. The users can step away from the values in Table 3.2 if they have better insight into their members' risk preferences.
- The seventh to eleventh parameters in Table 3.2 describe member characteristics. The users should change these values based on available data of their members.
- The last two parameters in Table 3.2 are important components of the retirement strategies in Case 3 and Case 4. In this paper, we show results for a constant $ 43,372 p.a. real income target[3]. For Case 4 the annuitisation ratio can be any value between 0 and 1. We only show results for a 50% annuitisation strategy for illustration purposes.

[2]The Australian Life Tables 2010-12 can be downloaded via `http://www.aga.gov.au/publications/life_table_2010-12/`

[3]This is the consumption level of comfortable lifestyle provided by ASFA.

Table 3.2: Parameter values for preference and financial assets. Sources are cited in brackets.

Parameter	Explanation	Value	Source
r_f	Real risk-free rate	0.00%	Assumption
μ_R	Mean of real equity return	5.00%	Assumption
σ_R	Standard deviation of real equity return	15.00%	Assumption
ω_t	Proportion invested in equity	50%	Assumption
ρ	Risk aversion	8	Assumption
ϕ	Residual benefit motive strength	0.83 or 0	Lockwood (2014)
b_0	Initial wealth ($1,000)	500	Input
x	Retirement age	65	Input
-	Gender	Male	Input
-	Family situation	Single	Input
-	Home-ownership	Non-homeowner	Input
-	Income target (for Case 3 & 4)	$43,372	ASFA Comfortable
-	Annuitisation ratio (Case 4)	50%	Input

Parameter values for the Age Pension rules are listed in Table 3.3.

Table 3.3: Age Pension eligibility and payment rates as at 1 July 2017. Single and non-home owner rates are used. Information is sourced from the Australian Government Department of Human Service website.

Parameter	Explanation	Value
$\bar{P}$	Full Age Pension payment rate (p.a.)	$22,804.6
τ^A	Taper rate under the asset test	0.003
$\underline{b}$	Threshold for full pension under asset test	$450,500
$\underline{b}_1$	Threshold for different deeming rates under income test	$49,200
r_1	Lower deeming rate	1.75%
r_2	Higher deeming rate	3.25%
$\underline{I}$	Income test cut off point (p.a.)	$4,264
τ^I	Taper rate under the income test	0.5

3.3 Retirement Income and Residual Benefit

Figure 3.1 shows the average values and 90% confidence intervals of retirement incomes for the 4 cases.

The annual retirement income comes from three sources: draw-down from ABP, life annuity, and the Age Pension. Figure 3.2 compares the draw-down amount from ABP and the Age Pension amount across the four cases.

We decompose the mean retirement income and show the results in Figure 3.3.

The amounts of residual benefit for the four cases are compared in Figure 3.4.

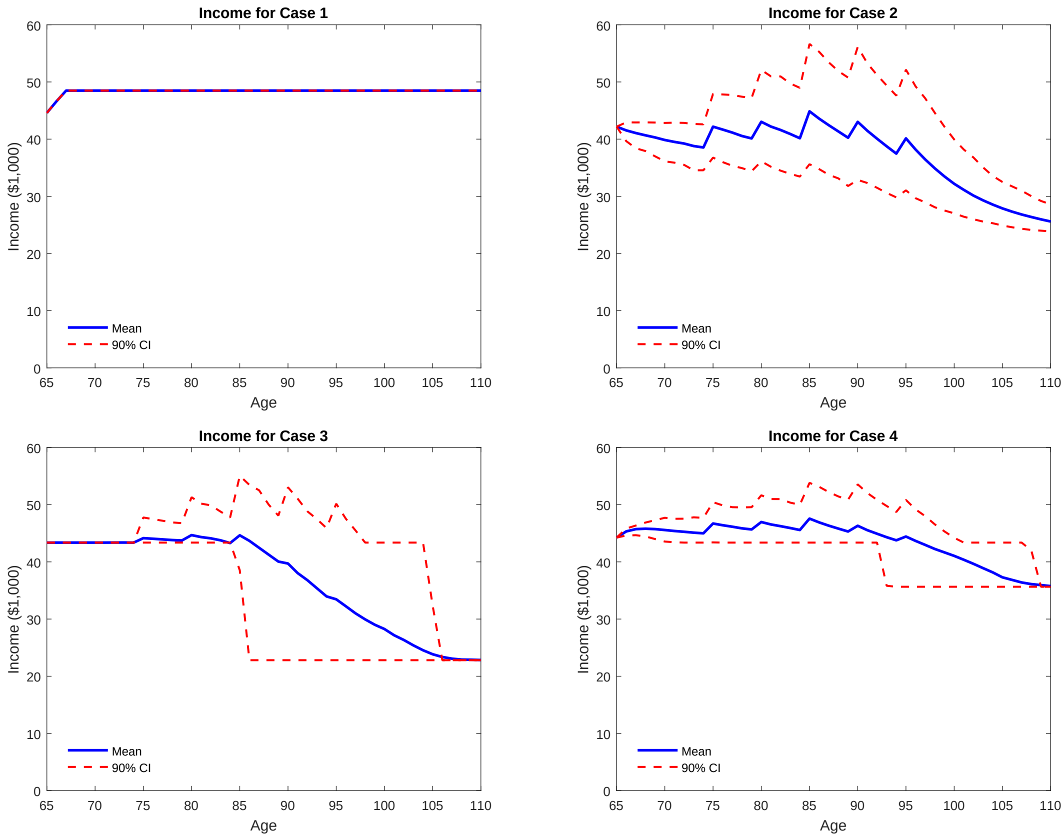

Figure 3.1: Retirement incomes for the 4 cases. Case 1 is where 100% assets are converted into life annuity; Case 2 is where the minimum drawdown rule is applied on ABP; Case 3 has a constant target income of $ 43,372 p.a. with only ABP; Case 4 has a constant target income of $ 43,372 p.a. with 50% ABP and 50% life annuity.

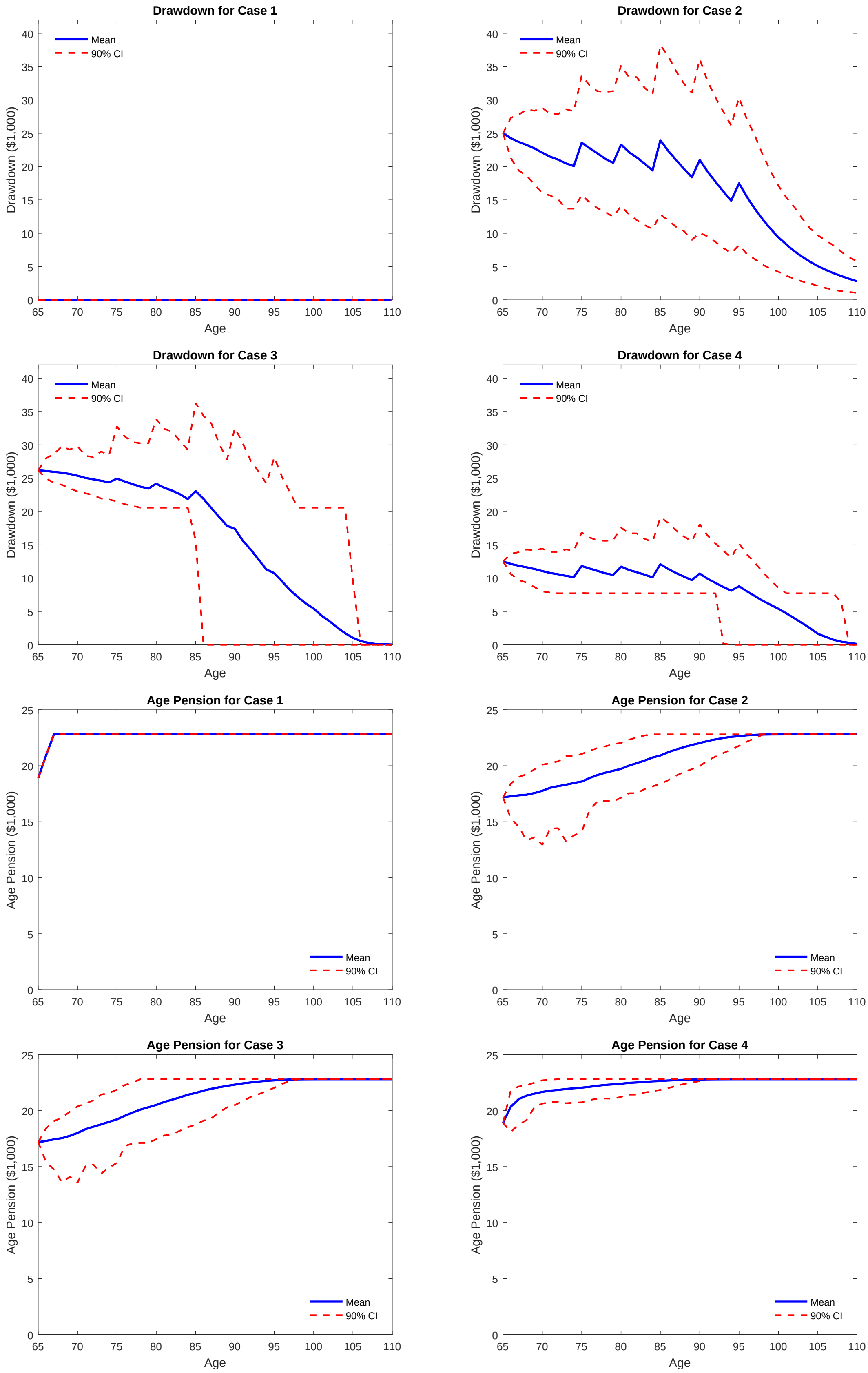

Figure 3.2: Draw-down from ABP and eligible Age Pension entitlements for the 4 cases. Case 1 is where 100% assets are converted into life annuity; Case 2 is where the minimum drawdown rule is applied on ABP; Case 3 has a constant target income of $43,372 p.a. with only ABP; Case 4 has a constant target income of $43,372 p.a. with 50% ABP and 50% life annuity.

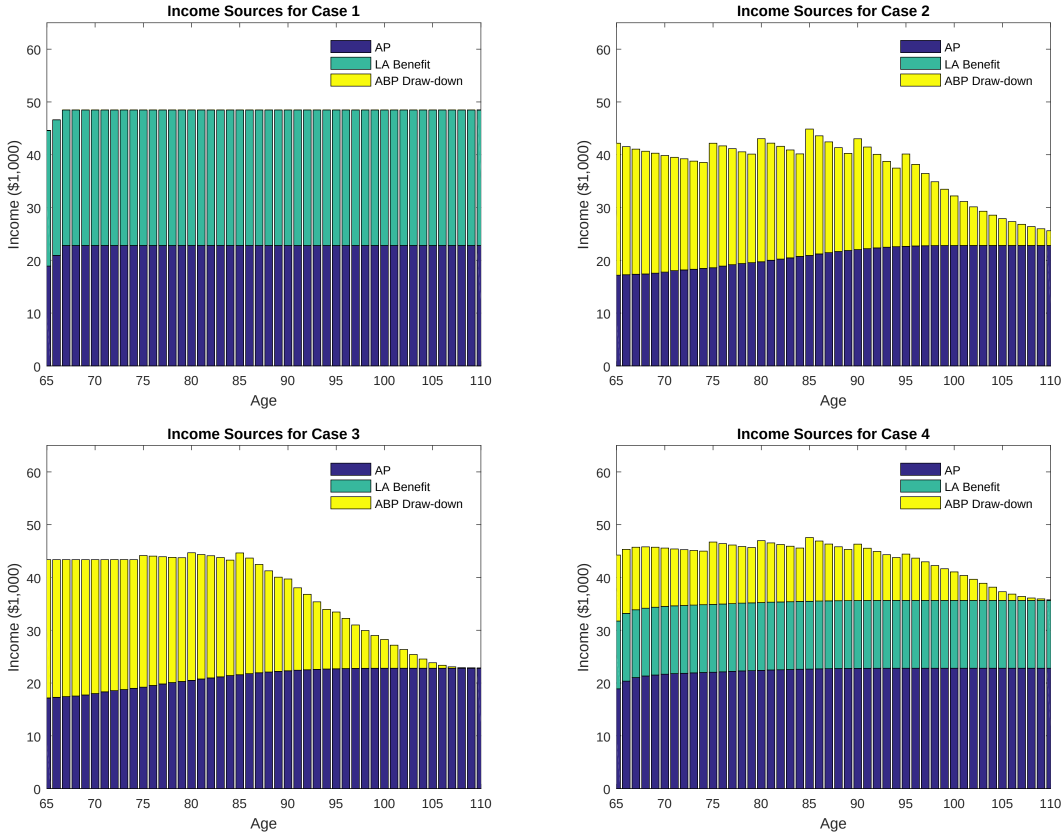

Figure 3.3: Decomposition of average retirement incomes for the 4 cases. Case 1 is where 100% assets are converted into life annuity; Case 2 is where the minimum drawdown rule is applied on ABP; Case 3 has a constant target income of $43,372 p.a. with only ABP; Case 4 has a constant target income of $43,372 p.a. with 50% ABP and 50% life annuity.

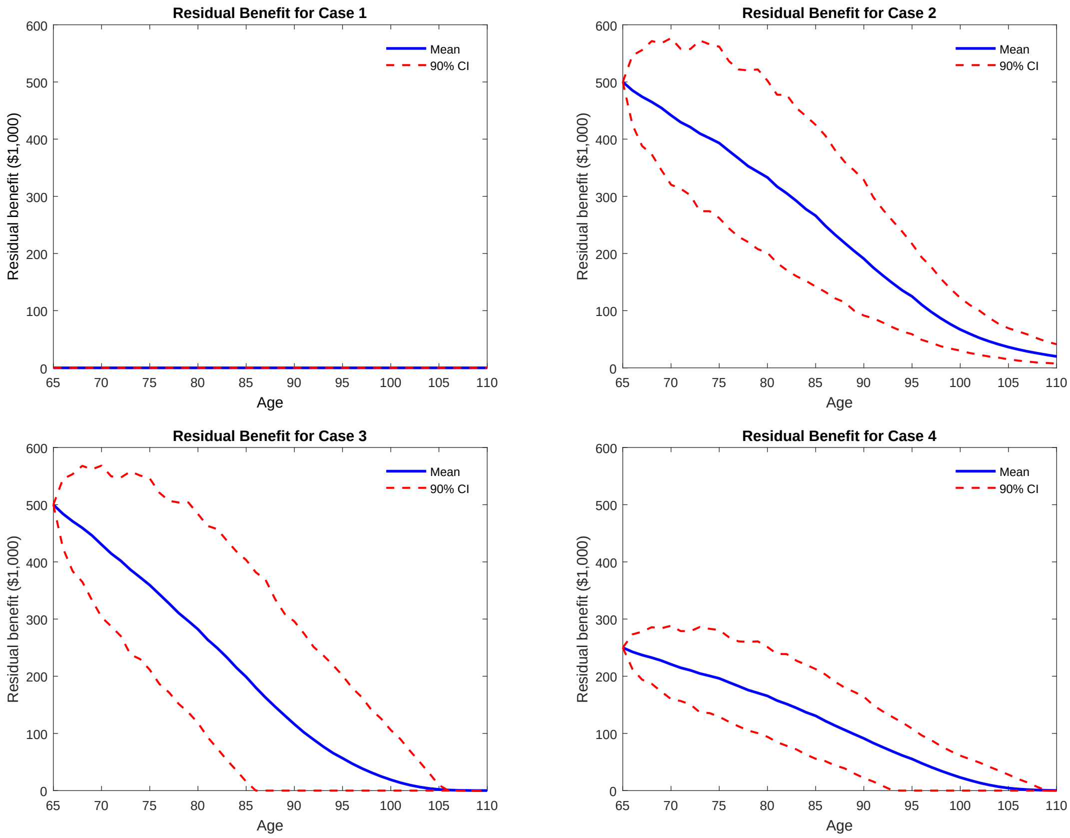

Figure 3.4: Residual benefit for the 4 cases. Case 1 is where 100% assets are converted into life annuity; Case 2 is where the minimum drawdown rule is applied on ABP; Case 3 has a constant target income of $43,372 p.a. with only ABP; Case 4 has a constant target income of $43,372 p.a. with 50% ABP and 50% life annuity.

3.4 Ranking of Expected Lifetime Utility

We investigate the ranking of these four cases in terms of the expected lifetime utility generated from these strategies.

3.4.1 With Residual Benefit Motive

Following Lockwood (2014), we set $\phi = 0.83$ in this scenario. See detail in "Technical Paper No.1: MDUF v1 Design".

The expected lifetime utility scores for the four cases are shown in the top panel of Table 3.4. Note we seek for the highest expected lifetime utility, which means we search for a solution that creates the smallest negative expected lifetime utility (which also results in the highest level of CEC). The CEC level generated in Case 2 is substantially higher than those in other Cases. Similar to comparing the expected lifetime utility, the absolute differences in CEC levels across different cases do not have important meanings, but rather the ranking is an important indicator in our comparison.

Table 3.4: Expected lifetime utility scores for the four cases.

Ranking	**Case**	**Expected Lifetime Utility**	**CEC ($)**
With residual benefit motive ($\phi = 0.83$)			
1	Case 2	-1.5042×10^{-26}	3,700.2
2	Case 4	-1.1213×10^{3}	0.2777
3	Case 3	-6.2396×10^{3}	0.2173
4	Case 1	-4.6125×10^{4}	0.1633
Without residual benefit motive ($\phi = 0$)			
1	Case 1	-0.4721×10^{-32}	31,425
2	Case 4	-0.7188×10^{-32}	29,593
3	Case 2	-1.9467×10^{-32}	25,667
4	Case 3	-4.0557×10^{-32}	23,112

The results show that the existence of a reasonable residual benefit motive makes the full annuitisation strategy (Case 1) the least favourable. Among the four solutions, the minimum drawdown rule (Case 2) delivers the best retirement outcome, as other solutions would have the risk of leaving too little in the residual account. Comparing Case 4 and Case 3, we see that annuitising 50% of wealth increases the expected lifetime utility. Comparing Case 1 with Case 3, we can see that Case 1 results in lower expected lifetime utility and therefore is an over-annuitisation strategy.

3.4.2 Without Residual Benefit Motive

We also investigate the performance of these different cases when the individual has no residual benefit motive. Results are shown in the bottom panel of Table 3.4. Note that leaving no or low residual benefit would result in a large negative lifetime utility if the residual benefit motive is positive. This is why the expected lifetime utilities for the same case with and without residual benefit motive, respectively, are very different.

It is observed that Case 1 where the asset is fully annuitised outperforms other strategies. The constant income target strategy on ABP (Case 3) delivers the worst outcome among the four strategies. Comparing Case 4 against Case 3, we observe that having some assets annuitised can make the strategy more attractive, whether there is positive residual benefit motive or not.

4 Conclusion

One important application of MDUF v1 is to compare different static retirement solutions. This paper provides technical detail in terms of model set-up as well as numerical examples of comparing four retirement solutions. The four solutions include a full annuitisation strategy, a minimum drawdown rule on ABP, a constant target income of $43,372 p.a. with only ABP, and a constant target income of $43,372 p.a. with 50% life annuity and 50% ABP.

This paper only focuses on an illustration of MDUF v1's application to comparing static solutions without extending to a wider range of all possible solutions. It is indeed not difficult to incorporate other static solutions. An interesting question is to find the best strategy that delivers the highest expected lifetime utility. Detail for this aspect is provided in "Technical Paper No.3: Optimal Dynamic Strategies".

References

Lockwood, L. M. (2014), Incidental bequests: Bequest motives and the choice to self-insure late-life risks, Technical report, National Bureau of Economic Research.